APPLICATION OF THE MONTE CARLO METHOD FOR CALCULATING THE SHAPE OF THE HEAT SOURCE GENERATED BY THE ACTION OF ELECTRON BEAMS ON A SUBSTANCE

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The space distribution of energy absorbed in a substance during the passage of an electron beam has been determined with the aid of the Monte Carlo method.

The problem in determining the shape of a heat source reduces to calculating the space distribution of energy absorbed in a substance during the passage of an electron beam.

In the Monte Carlo method of solution a complex stochastic process is treated as a sequence of elementary events. In view of the long machine time involved, however, in problems related to the transit of electrons the playout of parameters from the distributions characterizing a single interaction event is replaced by the playout of parameters from distributions describing multiple scatter [1-3].

The direction of electron travel is played out at the end of some interval taken from the Haudsmith – Saunderson distribution:

$$f_{\text{H-S}} = \sum_{l=0}^{n} \left(l + \frac{1}{2}\right) \exp\left[-G_l \cdot t\right] \mathscr{P}_l(\cos\theta), \tag{1}$$

$$G_l = \frac{0{,}0784\,(z+1)\,z}{A\beta^2\,(E_{\text{K}}^2 + 1{,}022E_{\text{K}})} \left\{P(-2,\ l) + \frac{\pi z\beta}{137\sqrt{2}} \cos\gamma P\left(-\frac{3}{2},\ l\right)\right\}. \tag{2}$$

The recurrence relations for calculating the functions $P(-2, l)$ and $P(-3/2, l)$ as well as values of $\cos\gamma = f(Z/137\beta)$ are given in [1, 4].

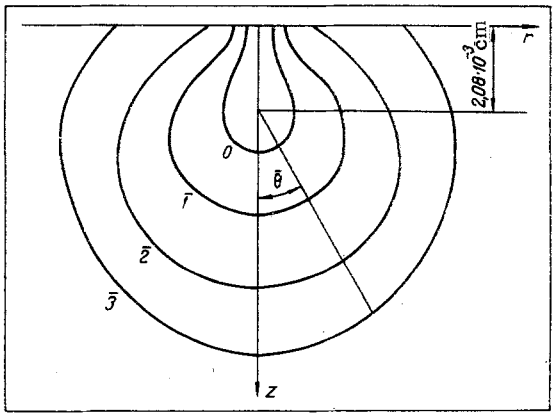

Fig. 1. Contour lines of equal energy losses $|-dE_K/dV|/|-dE_K/dV|_0$, $|-dE_K/dV|_0 = 10^6$ MeV/cm^3.

The t_i-interval is calculated by the Berger method [1] or is taken as 1/m-th of the total electron transit time (m denotes the number of intervals). The number of terms in series (1) is selected in accordance with the behavior of function $G_l(t)$ [5].

The energy of an electron at the end of the interval is calculated by the Bethe equation of continuous losses

$$\left|-\frac{dE_{\text{K}}}{dt}\right| = 0.1533\,\frac{z}{A\beta^2}\left\{\ln\frac{E_{\text{K}}^2\left(1+\sqrt{1-\beta^2}\right)}{2\sqrt{1-\beta^2}\,I^2(z)} + (1-\beta^2) - \ln 2\left[2\sqrt{1-\beta^2} - (1-\beta^2)\right] + \frac{1}{8}\left(1-\sqrt{1-\beta^2}\right)^2\right\}. \tag{3}$$

Considering the energy losses on plasma oscillations, we have

$$\left|-\frac{dE_{\text{K}}}{dt}\right| = 0.3066\,\frac{z}{A\beta^2}\ln 309.3\,\frac{\sqrt{E_{\text{K}}}}{\alpha}$$

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TABLE 1. Space Distribution of Energy Lost by an Electron in a Semiinfinite Aluminum Plate $|-dE_K/dV| \cdot 10^{-6}$ MeV/cm^3 (Initial electron energy E_0 = 128 keV, total transit range r_0 = 0.0104 cm)

z	r											
	0,1	1,2	2,3	3,4	4,5	5,6	6,7	7,8	8,9	9,10	10,11	11,12
0,1	9,28	0,15	0,13	0,09	0,08	0,052	0,05	0,04	0,03	0,02	0,01	0,01
1,2	9,83	0,84	0,20	0,15	0,12	0,1	0,08	0,05	0,04	0,03	0,02	0,02
2,3	8,17	0,91	0,33	0,22	0,17	0,129	0,10	0,08	0,05	0,04	0,03	0,03
3,4	5,1	1,61	0,52	0,30	0,22	0,161	0,12	0,10	0,06	0,05	0,03	0,03
4,5	3,08	1,5	0,68	0,37	0,24	0,187	0,14	0,10	0,08	0,06	0,03	0,04
5,6	1,64	1,18	0,67	0,43	0,30	0,216	0,15	0,12	0,08	0,06	0,04	0,04
6,7	1,15	0,86	0,54	0,40	0,28	0,211	0,16	0,12	0,08	0,06	0,04	0,03
7,8	0,71	0,63	0,5	0,37	0,27	0,204	0,16	0,11	0,07	0,05	0,04	0,03
8,9	0,52	0,45	0,4	0,34	0,27	0,209	0,15	0,11	0,08	0,05	0,03	0,02
9,10	0,46	0,39	0,32	0,29	0,23	0,196	0,14	0,09	0,07	0,04	0,02	0,01
10,11	0,3	0,31	0,28	0,22	0,21	0,156	0,11	0,08	0,05	0,04	0,01	0,01
11,12	0,26	0,23	0,21	0,17	0,16	0,114	0,09	0,06	0,04	0,02	0,01	0,00
12,13	0,21	0,19	0,15	0,14	0,12	0,077	0,06	0,03	0,02	0,01	0,00	0,00
13,14	0,08	0,1	0,10	0,10	0,06	0,05	0,03	0,02	0,01	0,00	0,00	0,00
14,15	0,03	0,06	0,05	0,05	0,04	0,027	0,01	0,00	0,00	0,00	0,00	0,00
15,16	0,05	0,04	0,03	0,02	0,02	0,012	0,04	0,001	0,00	0,00	0,00	0,00

$$+\frac{(\Delta E)^2}{\rho a_0 m_0 c^2 \left[1-\frac{m_0c^2}{E_K+m_0c^2}\right]} \ln\frac{3680\alpha\sqrt{E_K}}{\Delta E}, \qquad (4)$$

$$\alpha = k_c \cdot 10^{-8}.$$

As a result of calculations, we have obtained the distribution of energy losses in a semiinfinite aluminum plate for electrons with a 128 keV energy and normally incident at point (r = 0, z = 0). Numerical values of the distribution density MeV/cm^3 in space intervals [jd < z < (j + 1)d, id < r < (i + 1)d], d = r_0/20 are given in Table 1 without losses on plasma oscillation and in Table 2 with losses on plasma oscillation taken into account.

It is to be noted that the main source of error in this method of calculation lies in the use of the Bethe equation (3) for determining the energy losses per interval. Formula (4) agrees with the experiment by 5-10% better and its use improves the accuracy of calculations.

A distribution corresponding to the data in Table 2 is shown in Fig. 1. The equipotential lines plotted to a logarithmic scale illustrate the tendency of the distribution to taper off exponentially with increasing distance from the maximum-density point in a $\bar{\theta} < 90°$ direction (approximate value of the exponent is here 641). As the surface is approached, the distribution deviates more from an exponential one.

TABLE 2. Space Distribution of Energy Lost by an Electron in a Semiinfinite Aluminum Plate $|-dE_K/dV| \cdot 10^{-6}$ MeV/cm^3, with Losses on Plasma Oscillation Taken into Account (E_0 = 128 keV, r_0 = 0.0135 cm)

z	r											
	0,1	1,2	2,3	3,4	4,5	5,6	6,7	7,8	8,9	9,10	10,11	11,12
0,1	5,20	0,09	0,06	0,05	0,05	0,03	0,02	0,02	0,01	0,01	0,00	0,00
1,2	4,29	0,26	0,12	0,10	0,07	0,05	0,04	0,04	0,02	0,01	0,01	0,00
2,3	3,09	0,60	0,20	0,14	0,10	0,06	0,05	0,04	0,03	0,01	0,01	0,01
3,4	1,90	0,77	0,31	0,17	0,12	0,08	0,00	0,05	0,04	0,02	0,01	0,01
4,5	0,88	0,58	0,33	0,19	0,13	0,09	0,07	0,05	0,04	0,03	0,01	0,01
5,6	0,56	0,43	0,30	0,19	0,13	0,09	0,07	0,05	0,04	0,02	0,01	0,01
6,7	0,39	0,32	0,26	0,20	0,15	0,09	0,06	0,05	0,04	0,02	0,01	0,01
7,8	0,26	0,24	0,20	0,16	0,12	0,09	0,05	0,05	0,03	0,02	0,01	0,00
8,9	0,17	0,17	0,16	0,12	0,11	0,07	0,05	0,05	0,03	0,02	0,01	0,00
9,10	0,11	0,12	0,11	0,11	0,08	0,05	0,04	0,03	0,02	0,01	0,00	0,00
10,11	0,07	0,9	0,09	0,07	0,06	0,05	0,03	0,02	0,01	0,01	0,00	0,00
11,12	0,05	0,6	0,06	0,05	0,04	0,03	0,02	0,02	0,01	0,00	0,00	0,00
12,13	0,05	0,5	0,03	0,03	0,02	0,01	0,01	0,00	0,00	0,00	0,00	0,00
13,14	0,04	0,2	0,01	0,01	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00
14,15	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
15,16	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00

Calculations were made on a Minsk-22 computer. A total of 500 games was played out. The computer time was 3 h. The accuracy of this calculation was the same as that of analogous one-dimensional solutions to (1)-(3), where the nature and magnitude of errors were analyzed.

NOTATION

f_{H-S}	is the density of Haudsmith–Saunderson distribution;
$\mathscr{P}_e(\cos\theta)$	are Legendre polynomials;
z, A	are the atomic number and atomic weight of an element;
$\beta = v/c$	is the ratio of electron velocity to velocity of light;
E_K	is the kinetic energy of an electron, MeV;
I(z)	is the mean ionization potential of atoms in the medium;
ΔE	is the plasmon energy;
k_c	is the wavelength corresponding to effective interception;
a_0	is the Bohr radius;
m_0	is the quiescent mass of an electron;
θ	is the deflection angle of electron within an interval.

LITERATURE CITED

1. M. I. Berger, NBS Tech. Note No. 187 (1963).
2. H. E. Bishop, Proc. Phys. Soc., 85, No. 547 (1965).
3. A. F. Akkerman, Yu. A. Andreev, and Yu. G. Lyutov, Fiz. Tverd. Tela, 9, 766 (1967).
4. L. V. Spencer, Phys. Rev., 98, No. 6 (1955).
5. G. E. Gorelik and S. G. Rozin, Inzh.-Fiz. Zh., 21, No. 2 (1971).